Sub-diffusive and Sub-advective Model for Fluid Flows in Porous/Debris Media: Some Exact and Numerical Solutions

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Abstract: In geophysical and industrial mass flow processes, both the solid and fluid phases may be involved. By neglecting the solid deformation and inertial forces in the two-phase physical-mathematical model for debris flow (Pudasaini, 2012), a novel sub-diffusive and sub-advective (SD-SA) equation for fluid flows through general porous landscape and debris material is obtained (Pudasaini, 2014). In general, it is not possible to find exact analytical solutions to the full SD-SA model. However, some exact solutions for the reduced sub-diffusion process are constructed. We employ special mathematical techniques, mainly, transformation to classical advection-diffusion equation, separation of variables and linearization, to further construct analytical solutions for the SD-SA model (Khattri, 2014). Some advanced exact solutions are also constructed by using Bring ultra-radical and hyper-geometric functions by generating the Abel and Lineard canonical equations (Khattri, 2014; Pudasaini, 2014). The full SD-SA equations are integrated numerically by applying the high-resolution TVD-NOC schemes. Our numerical solutions match exactly with the exact analytical solutions for sub-diffusion, thereby demonstrating the very high accuracy and performance of the numerical method and computational code. It is revealed that the sub-diffusion and sub-advection processes (as characterized by higher flux exponents) are fundamentally different as compared to the classical diffusion-advection, and that the physics of fluid flows in debris and porous landscape is better described by the sub-diffusion and sub-advection model. The results, their underlying physics and applications may help in constructing early warning and mitigation strategies in potentially catastrophic failures of landslides, reservoir dams and embankments in geo-disaster-prone areas.

Keywords: Flow in porous media, Two-phase flows, Sub-diffusive and sub-advective equation, Exact solutions, High-resolution numerical simulations.

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