Velocity measurements in dry granular avalanches using particle image velocimetry technique and comparison with theoretical predictions

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Velocity and depth are crucial field variables to describe the dynamics of avalanches of sand or soil or snow and to draw conclusions about their flow behavior. In this paper we present new results about velocity measurements in granular laboratory avalanches and their comparison with theoretical predictions. Particle image velocimetry measurement technique is introduced and used to measure the dynamics of the velocity distribution of free surface and unsteady flows of avalanches of non-transparent quartz particles down a curved chute merging into a horizontal plane from initiation to the runout zone. Velocity distributions at the free surface are determined and in one case also at the bottom from below. Also measured is the settlement of the avalanche in the deposit. For the theoretical prediction we consider the model equations proposed by Pudasaini and Hutter [J. Fluid Mech. 495, 193 (2003)]. A nonoscillatory central differencing total variation diminishing scheme is implemented to integrate these model equations. It is demonstrated that the theory, numerics, and experimental observations are in excellent agreement. These results can be applied to estimate impact pressures exerted by avalanches on defence structures and infrastructures along the channel and in runout zones. © 2005 American Institute of Physics. [DOI: 10.1063/1.2007487]

I. INTRODUCTION

The most important physical quantities of avalanche dynamics are probably the velocity distribution and evolution of the avalanche boundary from its initiation to the deposit in the runout zone and the depth profile of the deposit. Velocity and depth are crucial to describe the dynamics and to draw inferences about the flow behavior of an avalanche. However, velocity measurements and their direct comparison with theoretical predictions for free-surface flow avalanches are still lacking in the literature. Our aim in this paper is to implement the particle image velocimetry (PIV) measurement technique for the measurements of the dynamics of the velocity field of unconfined and free-surface flows of granular avalanches, and to check the validity of the extended Savage-Hutter model in the prediction of both the velocity field and the geometry of the sliding and deforming granular body.

From a structural engineering and planning point of view one must properly predict the velocity field of a possible avalanche in order to adequately design buildings, roadways, and rail transportations in mountainous regions and appropriately estimate impact pressures on obstructing buildings that may be hit by an avalanche along its track down a mountain valley. Equally important is to know the velocity field of flowing granular materials and fine granulates through various channels in process engineering scenarios. Thus, in order to acquire confidence in the model equations it is vital to corroborate them by direct observation. In this spirit, we performed several laboratory experiments with quartz particles in order to check the validity of the theory. We used a modern optical measurement technique, particle image velocimetry, to measure the velocity field of the particles at the free surface and at the bottom in an unsteady motion of an unconfined avalanche over a chute that is curved in the main flow direction and merges continuously into the horizontal runout zone. We presented the results for different regions of the chute and for different times.

In particular, we are also interested to acquire knowledge of the depth profile of the deposit of the avalanche in the runout zone. Its correct determination is very important in real applications at least from two aspects: (i) When knowing the actual runout distance (area) and the distribution of the height of the deposit, one can easily divide avalanche-prone mountain terrains and valleys into three zones: (a) avalanche danger zone, (b) less dangerous zone, and (c) danger-free zone, i.e., one can construct the hazard map of the specific site. This division is quite standard in many countries. (ii) Alternatively, if there is good agreement between theory and

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experimental measurement of the height profile in the deposit, then one can easily infer reliability and efficiency of the theory over the entire avalanche path. In this way the theory can be used to predict the velocity distribution, depth profile, impact pressure, strain rate, and other relevant quantities as the avalanche slides down the mountain side. For these reasons, we have very carefully measured the depth profile of the deposit of the avalanche with a penetrometer. Agreement on the laboratory scale paired with the scale invariance of the problem brings confidence on the large-scale behavior.

We are looking for correspondence and harmony between the Savage-Hutter avalanche theory, the adequacy of the numerics, and experimental findings. The model equations employed here are extensions of the Savage-Hutter theory by Pudasaini and Hutter\textsuperscript{1} and Gray et al.\textsuperscript{2} These equations, which describe the distribution of the avalanche thickness and the topography-parallel depth-averaged velocity components, are a set of hyperbolic partial differential equations for generally curved and twisted channels. The model equations are numerically solved by implementing the Nonoscillatory central (NOC) differencing scheme with total variation diminishing (TVD) limiters, see, e.g., LeVeque.\textsuperscript{3} These are high-resolution numerical techniques that are able to resolve steep heights and velocity gradients and moving fronts often observed in experiments and field events but not captured by traditional finite difference schemes.

We will present the experimental and theoretical results on both the velocity and height of the free-surface flow of avalanches and compare the experimental findings against theoretical predictions. Although some fundamental researches have been carried out in this direction,\textsuperscript{4-6} to our knowledge, such results have not been presented before (in such detail by making direct comparisons with the theoretical predictions) in the literature of the dynamics of debris flows and avalanches. We are able to demonstrate that there is excellent agreement between theoretical predictions of model equations and experimental measurements. This, ultimately, proves the applicability of the theory and efficiency of the employed numerical method and establishes a nice and strong correlation among the theory, numerics, and experiments.

II. MODEL EQUATIONS

Let us consider the model equations proposed by Pudasaini and Hutter\textsuperscript{1} as an extension of the Savage-Hutter\textsuperscript{7} model for avalanching granular materials down general mountain slopes. The final thickness-averaged nondimensional balance laws of mass and momentum in slope-fitted curvilinear coordinates of mountain surfaces in the down-slope and cross-slope directions take the forms

\[
\frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} + \frac{\partial (hv)}{\partial y} = 0, \tag{1}
\]

\[
\frac{\partial (hu)}{\partial t} + \frac{\partial (h u^2)}{\partial x} + \frac{\partial (h u v)}{\partial y} = h s_x - \frac{\partial}{\partial x} \left( \frac{\beta_x h^2}{2} \right), \tag{2}
\]

where \( h \) is the depth of the avalanche, \( u, v \) are the depth-averaged velocity components parallel to the reference surface, and the factors \( \beta_x \) and \( \beta_y \) are defined, respectively, as

\[
\beta_x = -e g_x K_x, \quad \beta_y = -e g_y K_y, \tag{4}
\]

where \( e = H/L \) is the aspect ratio between typical avalanche height and its extent parallel to the bed. \( K_s \) and \( K_y \) in (4) are called earth pressure coefficients. Elementary geometrical arguments and Mohr’s circles may be used to determine these values as functions of the internal (\( \phi \)) and basal (\( \delta \)) angles of friction determined by (Hutter et al.\textsuperscript{8})

\[
K_x = K_{s_{\text{act/pass}}} = 2 \sec \phi (1 + \sqrt{1 - \cos^2 \phi \sec^2 \delta}) - 1, \quad (\partial u/\partial x) \equiv 0, \tag{5}
\]

\[
K_y = K_{s_{\text{act/pass}}} = \frac{1}{2} [K_x + 1 + \sqrt{(K_x - 1)^2 + 4 \tan^2 \delta}], \quad (\partial v/\partial y) \equiv 0,
\]

where \( K_x \) and \( K_y \) are active during dilatational motion (upper sign) and passive during compressional motion (lower sign).

The terms \( s_x \) and \( s_y \) represent the net driving accelerations in the down-slope and cross-slope directions, respectively, and are given by

\[
s_x = g_x - \frac{u}{|u|} \tan \delta (-g_x + \lambda \kappa u^2) + e g_x \frac{\partial b}{\partial x}, \tag{6}
\]

\[
s_y = g_y - \frac{v}{|u|} \tan \delta (-g_y + \lambda \kappa v^2) + e g_y \frac{\partial b}{\partial y}, \tag{7}
\]

in which \( |u| = \sqrt{u^2 + v^2} \) is the magnitude of the velocity field tangential to the sliding surface and \( g_x, g_y, g_z \) are the components of the gravitational acceleration along the coordinate lines. Similarly, \( \lambda = L/R \) is a measure of the radius of curvature \( R \) of the bed with respect to the avalanche length \( L \), \( \lambda \kappa \) is the local radius of curvature of the thalweg, while \( \eta \) gives the accumulation of the torsion of the thalweg from an initial position. Note that, since the used experimental chute is curved only on the downhill direction and is torsion-free, for the present paper we set \( \eta = 1 \). With this, these model equations reduce to those proposed by Gray et al.\textsuperscript{7}

The first terms on the right-hand side of (6) and (7) are due to the gravitational accelerations in the down- and cross-slope directions, respectively. The second terms emerge from the dry Coulomb friction and the third terms are projections of the topographic variations along the normal direction.

Given basal topography \( b \) and material parameters \( \delta \) and \( \phi \), Eqs. (1)–(3) allow \( h, u, \) and \( v \) to be computed as functions of space and time once appropriate initial and boundary conditions are prescribed. The former is a start from rest of material kept by spherical caps, the latter requires \( h = 0 \) at the avalanche margins.
III. NUMERICAL TECHNIQUES

The governing Eqs. (1)–(3) presented in Sec. II comprise a hyperbolic system in three variables, the avalanche thickness, and velocity components in the down-slope and cross-slope directions. Numerical schemes solving these equations must be able to grasp the typical behavior of such granular flows. Shock formation is an essential mechanism in granular flows on an inclined surface merging into a horizontal runout zone or encountering an obstacle when the velocity becomes subcritical from its supercritical state. It is therefore natural to employ conservative high-resolution numerical techniques that are able to resolve the steep gradients and moving fronts often observed in experiments and field events but not captured by traditional finite difference schemes. The NOC differencing scheme with the Minmod TVD limiter demonstrates the best numerical performance for simulating avalanche dynamics (see, e.g., LeVeque,\textsuperscript{3} but more particularly Pudasaini,\textsuperscript{9} Wang et al.,\textsuperscript{10} and Pudasaini et al.\textsuperscript{11}). The model Eqs. (1)–(3) can be rewritten in conservative form as

$$\frac{\partial w}{\partial t} + \frac{\partial f(w)}{\partial x} + \frac{\partial g(w)}{\partial y} = s,$$

(8)

where \( w = (h, m_x, m_y)^T \) is the vector of conservative variables of avalanche thickness \( h \) and depth-integrated down- and cross-slope momenta, \( m_x = hu \) and \( m_y = hv \), respectively. The down-slope and cross-slope momentum flux vectors \( f \) and \( g \) and the vector of the source terms \( s \) are given by

$$f = \begin{pmatrix} m_x \\ m_x^2/h + \beta_x h^2/2 \\ m_x m_y/h \\ m_y^2/h + \beta_y h^2/2 \end{pmatrix}, \quad g = \begin{pmatrix} m_y \\ m_x m_y/h \end{pmatrix},$$

$$s = \begin{pmatrix} hs_x \\ hs_y \end{pmatrix}.$$

(9)

The terms \( \beta_x \) and \( \beta_y \), defined in (4), incorporate the extending and contracting states of the avalanching mass through the active and passive earth pressures. Similarly, the source terms \( s_x \) and \( s_y \), described in (6) and (7), are of crucial importance as they include the total driving force generated by the gravity, friction, curvature, torsion, and local details of the basal topography. They jointly determine the dynamics of the flow.

The boundary of the computational domain is considered to be away from the physical margin of the avalanche. Along this boundary, the velocity and height, and their gradients, are set to zero.

IV. EXPERIMENTAL TECHNIQUE AND LABORATORY CIRCUMSTANCES

Data on measured velocity fields are scarce for both field and laboratory experiments. Velocity profiles were measured at selected points in real, large scale but artificially released avalanches by Dent et al.\textsuperscript{12} Gubler\textsuperscript{13} used radar Doppler measurements to determine the depth variation of the down-hill velocity in an artificially released flow avalanche. By using the PIV-measuring technique Eckart et al.\textsuperscript{14} measured the velocity field at the free surface and the bed of uniformly and steadily flowing fine granular masses (sand) in a rectangular two-dimensional (narrow) channel where the mass was continuously and uniformly discharged from a silo. No measurements were made for the overall free-surface motion of an avalanche from initiation to runout that was compared with theoretical predictions of some model equations. In this regard, the main goals of this paper are (i) corroboration of the physical adequacy of the model equations of the Savage-Hutter-type, and (ii) efficiency of the (NOC-TVD) numerical method by comparing it with corresponding results obtained by (laboratory) experiments performed under essentially well controllable circumstances together with an advanced measurement technique. The particle image velocimetry (PIV)-measurement technique is used to measure the dynamics of the velocity distribution of free-surface flows of avalanches down a curved chute merging into a horizontal plane. To our knowledge, such detailed velocity measurements were not performed so far.

A. Particle image velocimetry and technical details

We made use of the experimental method, called particle image velocimetry (PIV) technique, with which the velocity field of the free surface and basal boundary in a granular avalanche can be measured. This is an optical measuring system. The basic idea of this system is as follows: A part of the surface (or boundary) of a flowing granular avalanche, called the region of interest, is illuminated twice at time \( t \) and time \( t + \Delta t \). Then the two images (called “frame A” and “frame B”) are captured by a charge-coupled devices (CCD) camera. Displacements of identifiable particles of the moving avalanche are calculated by comparing frames A and B via pattern recognition. The displacement divided by the time difference \( \Delta t \) between both frames gives the velocity of a particle. We implemented a special PIV system, particularly designed for nontransparent fluids such as sand, gravel, quartz, or other granular materials that we can find in geophysical scenarios. More details on how it works can be found in Keane and Adrian\textsuperscript{15} and TSI’s manual for PIV (http://www.tsi.com/) for transparent fluids, and in Pudasaini\textsuperscript{9} and Eckart et al.\textsuperscript{14} for nontransparent fluids, particularly for granular flows and avalanches. The PIV system we use is called the “granular PIV.”

It is perhaps appropriate to describe the technical details of the electronic and optical equipments that were employed in the experiments. We use a system of the company TSI. It includes two CCD cameras of type TSI PIVCAM 13-8. The system is supplemented by super-wide-angle-zoom lenses of type NIKON NIKKOR AF 18–35 mm f/3.5 Gubler\textsuperscript{13} used radar Doppler system in three variables, the avalanche thickness /H20849 using the PIV-measuring technique Eckart hill velocity in an artificially released flow avalanche. By 093301-3 Velocity measurements in dry granular avalanches Phys. Fluids 17, 093301 (2005)
The highest temporal resolution is four double frames per second. The time delay between the first and the second frame can be set, which, of course, should depend on the range of the velocity values. We have chosen it to be in the order of 1 μs.

B. Chute geometry and arrangement

We performed several series of experiments over a chute as shown in Fig. 1. The chute is made of 3.5-mm-thick Plexiglas. It consists of three different parts connected together. The inclination angle of the upper inclined plane portion is 45°, it merges continuously into the horizontal runout zone. Specifically, the details are as follows: (i) upper inclined zone: length of 1560 mm, (ii) middle continuous transition zone: (curved) length of 370 mm, (iii) lower horizontal runout zone: length of 2250 mm. The width of all three parts is 1600 mm. Experimental images are taken separately in all three zones by repeating the experiments under essentially identical conditions and laboratory circumstances to assure the quality and reliability of the measurements. It should also be noted that if one includes the entire flow region (i.e., the whole chute) in a single capture the image and the data will be highly distorted because the chute is very large. In the upper right part of the chute, an electric analogue clock is mounted with two arms: the long arm performs one complete...
revolutions in 1 s, whereas one unit of the short arm stands for 1 s. In the middle of the top of the inclined portion a cap, cut from a sphere, is mounted to hold the initial mass of the granules. The cap is made of Plexiglas with the supporting frame made of steel which can instantly be lifted. This motion in the opening process is a rotation about a horizontal axis. The opening of the cap, flashes, cameras, clock, and PC are all synchronized.

C. Initial conditions and length scales

The upper part, above 30° latitude, of a hemispherical cap of radius 195 mm is cut and used as the cup to hold the initial pile of the granular material, see Fig. 1. This design of the cap prevents the granules from a free fall motion at the front and the top of the heap at the time of release. Furthermore, with this choice, the rear part of the heap does not initially move backward. Therefore, we can define the initial computational velocity to be \((u_0,v_0)=(0,0)\). 8.7 kg mass of quartz particles of 5-mm nominal diameter are used for the experiments and the simulations. The internal and the bedding angles of this material have independently been measured and are \(\phi=35°\) and \(\delta=23°\), respectively, with errors of no more than \(\pm2°\).

The model equations contain two nondimensional parameters \(\varepsilon\) and \(\lambda\). They are associated with the geometry of the avalanching material and the chute. Provided that the geometry is similar these equations predict the same avalanche flow irrespective of whether it is in a small-scale laboratory run or a geophysical avalanche in a large-scale mountain environment. To achieve possibly greater generality and a real feeling both the experimental and the computed results are presented in dimensional variables. The appropriate physical variables for a particular application can be chosen by applying the (back) transformation of the scalings (Pudasaini and Hutter). We chose the same length scale in all \(x, y, z\) directions as \(L=H=10\) cm and \(R=10\) cm providing \(\varepsilon=H/L=1\) and \(\lambda=L/R=1\). This preserves the aspect ratio of the physical avalanche and makes it easier to interpret the results without any distortion of geometry and velocity obtained from the computation of the model equations.

V. VALIDATION OF THE THEORY

One of our major aims is the validation of the extended Savage-Hutter theory by some laboratory experiments. Particularly, we will demonstrate how good the agreement can be between the theoretical prediction of the velocity distribution of an unsteady flow avalanche down a curved chute and the measurement results by the PIV technique.

A. Experiments using quartz particles

Figure 1 depicts a series of experimental snapshots of an avalanche in the laboratory taken from a digital video (DV) camera. The bulk material of quartz granulates is held by the cap. The first panel shows a photograph before lifting the cap defining the initial condition of the avalanche, the second panel describes the circumstances right after opening. As soon as the cap is removed the bulk material is continuously extending mostly in the direction of steepest descent. The third panel captures the fully developed flow in which the entire granular mass lies in the upper inclined zone. Comparing the first and second panels one can see that the front of the avalanche accelerates faster than its tail. The reason for this is that the entire heap is under a passive state of stress before the cap is lifted. Immediately after the release of the cap the front part of the heap no longer suffers a surface stress from the confining cap, and an active state of earth pressure is established. The remaining grains still feel the stresses from their neighbors (which is passive) until the wave front that separates the active from the passive states has reached the upper part. So, the motion of the frontal part of the pile is ahead of that in the rear portion. Moreover, the initial surface slope triggers the downhill motion, whereas that in the rear part acts in the uphill direction. In the fourth panel, the avalanche front has entered the transition zone. The front of the avalanche has just crossed the front boundary of the transition to the deposition zone. Due to the downslope curvature of the chute topography in the transition zone the avalanching front starts decelerating. Therefore, the mass in the front is contracting in the downhill direction due to the effect of the passive earth pressure, but the mass in the tail is still extending. The deposition of the mass commences in the vicinity of the lower end of the transition zone. The fifth panel shows a picture of the avalanche at a time when the major part of the body lies in the runout zone and the body is approaching its rest position. The rear material is now hindered from freely moving forward, it dilates now in the cross-slope direction and begins to broaden its deposition. The far end part of the tail consists mainly of fine granulates and the powder, which is an inevitable constituent of the bulk material. The final panel shows the deposit of the avalanche which lies entirely in the horizontal runout zone of the chute. Although, immediately before the deposit, the front of the body is almost at standstill the mass from the tail is still flowing down and deposited on the tail side of the body. A steep surface-height gradient is thus developed on the tail side of the avalanche. Occasionally, this steep backward slope is slightly weakened in the last phase of the motion by a backward motion of the top granules to reestablish the local angle of repose. The deposit is of convex shape, more or less ellipsoidal with the major axis along the lateral direction. Actually, in all panels the flowing granular body is fairly compact with only slightly diffusive margins due to particle bouncing so that the continuum assumption seems to be justified. The shape of the body depends on the material properties, i.e., internal and bed friction angles, the chute geometry, the geometry of the material in its initial position, and the initial conditions. The motion of the bulk and the deforming body from the first panel to the sixth panel defines the complete dynamics of the avalanche as a rapid free-surface motion of dry granular material from initiation to deposit.

B. PIV measurement and validation of the theory

Before presenting a detailed comparison between theory and measurements, some remarks are in order. In fact, ensu-
FIG. 2. (Color). Comparison between pile geometries and the velocity distributions at the surface according to the theoretical prediction (left panels) and the PIV measurements (right panels). The experimental configuration is explained and presented in Fig. 1. Very good agreement between theory and experimental measurements of the velocity distribution is seen.
ing figures show the full frames captured by the camera(s) (see panels on the right of Fig. 2). This implies that consecutive pictures correspond to different experiments, repeated under identical external conditions. Also note that the different colors in these pictures represent the magnitudes of a kind of mean velocity distribution in the corresponding regions determined by the contours computed from the real data of the velocity magnitudes. The magnitudes of the velocity fields are shown in the right color bars of each picture. It should therefore be clear that the real range of the actual velocity field may be somewhat larger than that presented here. As we increase the number of contours, the range of the color bars may equally increase. Furthermore, the arrows indicate the directions and their lengths represent the relative magnitudes of the actual velocity vectors in each panel.

Figure 2 depicts the comparison between the theoretical predictions (left panels) and PIV measurements (right panels) of an avalanche of quartz particles sliding and deforming down a curved Plexiglas chute as shown and explained in Fig. 1. The comparison is presented at five consecutive times as indicated in the upper left corners of each panel immediately after the onset of the motion of an avalanche until it comes almost to the deposit in the horizontal runout zone.

The two upper most panels of Fig. 2 present the theoretical versus the experimental results at time 0.38 s. The flow is fully developed, unsteady and the granular mass lies entirely on the inclined upper zone of the chute. The motion is mainly in the down hill direction (with some sidewise spreading), accelerating, and the velocity field is symmetric about the central line (y = 0) of the chute. The color bars on the right of each picture indicate the magnitudes of the velocity fields in ms\(^{-1}\). Differences are only seen in the curvature of the lines separating the differently colored velocity regions. Apart from this, comparison of the two panels shows excellent agreement between theory and experiment for both the geometry (boundary) and the velocity distribution of the avalanche of quartz particles for this time step.

The second row contains the theoretical and experimental results at time 0.63 s. A trace of a boundary layer effect along the margins can be seen in the experimental panel. As soon as the mass crosses the upper boundary of the transition zone (horizontal red line) the flow switches from its supercritical to the subcritical state, and the mass starts decelerating. This means that the flow speeds are above and below the critical to the subcritical state, and the mass starts decelerating. Since the applied technique is only apt for the velocity measurement we cannot use it to determine the three-dimensional velocity field. Moreover, the panels exhibit the motion of the avalanche in three parts: the upper inclined zone, the middle transition zone, and the lower runout zone. A close look at the lower parts of both panels reveals that the granular body is contracting at its front in the runout zone. Since the chute is laterally unconfined, the granular mass is extending in the cross-slope direction near the front. This spreading is symmetric about the central line. The longitudinal earth pressure is increasing (passive pressure state) and this information is propagating upstream. This can clearly be observed if we compare these panels with the panels of the third row because the magnitude of the velocity field has considerably decreased. On the other hand, the cross-slope earth pressure is decreasing (active pressure state). This information is also propagating upstream. As soon as the mass enters the runout zone the velocities of the particles decrease rapidly. Consequently, the mass is extending in the lateral direction. Although the rear margin is far more pointed in the experimental pile than in its computed counterpart (the source for the rounded tail is unknown and we can only speculate about the possible causes: local fines may reduce the bed friction angle; increased bed friction at the rear margin, which seems to be evident from the colors at the immediate rear end of the experimental avalanche), comparison of the theoretical prediction with the experimental result nevertheless still reveals very good agreement.

The last panel describes the state of the avalanching mass just before it comes to rest at the time 1.25 s. The entire mass of the body now lies in the horizontal runout zone. Although the particles close to the rear ends still have considerable velocity, those near the front of the body are close to rest. Note that, although the color distribution does not seem to correspond exactly to each other between theory and measurement, their numerical values agree quite well. The discrepancy in the shape of the pile body is also due to the contour plotting because it neglects that part of the body which has very small velocity magnitude. Apart from this, the lateral and longitudinal spreads of the body and velocity distributions between the theoretical predictions and the experimental measurements are both in very good agreement.

C. Evolution of the avalanche geometry

We have seen in Sec. V B that the PIV measurements can obviously be used to determine the avalanche boundary. Since the applied technique is only apt for the velocity measurements we cannot use it to determine the three-
dimensional evolution of the avalanche geometry. For this, other techniques such as digital photography, digital photogrammetry, laser sheets, or laser cartography must be used. One of the most important aspects in avalanche dynamics is the determination of the runout area and the height profile of the avalanche in its deposit. This is so because with this information we can construct the hazard map and estimate the impact pressures on the obstructions in the runout zone. The evolution of the three-dimensional geometry along the entire track is not so vital. For this reason we measured the avalanche height in the deposit using a penetrometer. Figure 3 displays the contours of the depth of the avalanche in the deposition area both for the theoretical prediction and the experimental measurement at times $\geq 1.35$ s when the avalanche was at rest. It is clearly seen that the lateral and the longitudinal runout distances, the over all runout zone, as well as the height profiles are rather well predicted by the theory.

**D. Multi-CCD cameras and velocity shearing**

The avalanche equations presented in Sec. II to predict the velocity and evolution of the avalanche geometry is based on realistic assumptions. One of them concerns the velocity distribution. We assumed that the velocity profile is almost uniform through the depth of the avalanche. This assumption may not be adequate right after the release, in the vicinity of obstructions and close to the deposit where both the height and vertical component of the velocity field could change considerably, in some cases even abruptly. Therefore,
assuming uniform velocity is a good approximation only when there is a smooth boundary. However, along the main flow path or along the track of the avalanche, the concept of a uniform velocity distribution through its depth should be a fairly reasonable assumption. We analyzed the images just before the transition zone so as to have both a fully developed unsteady flow and good quality and resolution of the images. Also note that, for simplicity, only a rectangular portion of the avalanche containing the central line of the chute is taken into account. Two cameras were placed and aligned parallel to the normal of the chute surface about 1000 mm distant from the chute on both sides. Since we were using a Plexiglas chute, capturing images from either side of the chute was possible. The measured velocity distributions from the top and bottom of the chute show that the difference between the top and bottom mean velocities is about 5%, providing the physical justification of the assumption on the velocity profile through the depth of the avalanche. We have computed the relative difference between the top and bottom velocities with the following expression:

$$\text{Relative difference} = \frac{\int_A (v_{\text{top}} - v_{\text{bottom}})dA}{\sqrt{\int_A v_{\text{top}}dA} \sqrt{\int_A v_{\text{bottom}}dA}},$$

where $A$ is the area of the image zone, $v_{\text{top}}$ and $v_{\text{bottom}}$ are the velocities at the top and bottom, respectively. The difference in the mean between the top and bottom velocity fields is computed by

$$\text{Relative difference in mean} = \frac{\bar{v}_{\text{top}} - \bar{v}_{\text{bottom}}}{\sqrt{\bar{v}_{\text{top}} \bar{v}_{\text{bottom}}}},$$

where $\bar{v}_{\text{top}}$ and $\bar{v}_{\text{bottom}}$ are the mean values of the velocities at the top and bottom, respectively.

Furthermore, in the sequel, we will also discuss standard deviations at the top and bottom of the flow, respectively.

Figure 4(a) depicts the velocity measurement at the free surface of a fully developed avalanche of quartz particles initially kept in a cut of the hemispherical cap as explained earlier. Similarly, Fig. 4(b) displays the velocity field at the bottom of the avalanche measured from the opposite side of the Plexiglas chute. To have better visualization this field is mirrored about the central line of the chute.

Figure 4. (Color) Velocity distribution (a) at the free surface and (b) at the bottom. The actual topographic location of the chute is shown in length units in mm. The left velocity field is computed from the image captured by the camera from the top (free surface) and the right field is its counterpart computed from the image captured simultaneously by another camera from the opposite side (bottom) of the Plexiglas chute. To have a better visualization the right field is mirrored about the central line of the chute.
metal bars (the supports of the chute). These reflections artificially increase the magnitudes of the velocity field. However, there is no problem of this kind for the image taken from the top of the chute. The higher value of the standard deviation (for the right figure) also manifests the random fluctuation of the velocity field due to these reflections. Otherwise, comparison between these two figures reveals that for the fully developed motion of an avalanche the velocity distribution through the depth is highly uniform.

VI. CONCLUDING REMARKS

One of the most basic and fundamental questions related to the avalanche theory presented in Sec. II is are these model equations really able to simultaneously predict flow properties like velocity and flow depth in chutes and channels? Several results can be found in the literature concerning the geometric deformation of the granular pile from initiation to deposit.1,2,17,18 However, comparison between theory and experimental results for both the dynamics of the velocity field and the geometry of the avalanche along the channel and in the runout zone could not be found in the existing literature. Such results are presented in this paper.

The velocity distribution and the evolution of the avalanche boundary from its initiation to the deposit on the runout zone and the depth profile of the deposit are measured. We introduced and used the particle image velocimetry (PIV)-measurement technique to measure the velocity field of the nontransparent granular particles at the surface and the bottom of completely free-surface flow of nonuniform and unsteady motions of avalanches over a chute that is curved in the main flow direction and merges continuously into the horizontal runout zone. The results are presented for different regions of the chute and for different times. We are able to demonstrate excellent agreement between the theoretical predictions and the experimental measurements. This, ultimately, proves applicability of the theory and efficiency of the numerical method and establishes a very good correlation between theory, numerics, and experiments.

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