Avalanching granular flows down curved and twisted channels: Theoretical and experimental results

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Depth evolution and final deposits play a crucial role in the description of the dynamics of granular avalanches. This paper presents new and important results on the geometric deformation and measurements of avalanche depositions in laboratory granular flows and their comparisons with theoretical predictions through some benchmark problems for flows down curved and twisted channels merging into a horizontal plane. XY-table and analog laser sensor are applied to measure geometries of deposited masses in the fanlike open transition and runout zones for different granular materials, different channel lengths, and different channel mouths in the runout zone. The model equations proposed by Pudasaini and Hutter1 (“Rapid shear flows of dry granular masses down curved and twisted channels,” J. Fluid Mech. 495, 193 (2003)) are used for theoretical prediction. We show that geometric parameters such as curvature, twist and local details of the channel play a crucial role in the description of avalanching debris and their deposits in the standstill. Asymmetric depositions and surface contours about the central line of the channel could not be produced and predicted by any other classical theories and available experiments in the literature as done in this paper. Such a role played by the geometrical parameters of the channel over physical parameters for the flow of granular materials down a general channel was not investigated before. It is demonstrated that the numerical simulations of the model equations and experimental observations are generally in good agreement. © 2008 American Institute of Physics. [DOI: 10.1063/1.2945304]

I. INTRODUCTION

There are two basic field variables, namely, the velocity and depth fields which describe the evolution of any avalanching granular flow.1−5 These are required informations to draw inferences about the flow behavior of granular avalanches of sand, soil, gravel, or other granular materials with ample application in process engineering and in geodynamics. The evolution of the avalanche geometry from its initiation to the deposit in the runout zone and the depth profile of the deposit are primary entities, along with the velocity distribution, sought in these applications.8–13 In this paper we will focus on flows down rather general curved and twisted channels. The reason for describing and analyzing flows through such general channel configurations is as follows: There might be many applications of channel flows in the transportation of granular materials and powder substances through curved and twisted channels in process engineering. As far as we know, this paper is a first attempt to address such a problem, where experimental results are presented for the first time and compared to the corresponding theoretical predictions associated with the natural and suitable channel configurations.

Pudasaini and Hutter1 proposed continuum dynamical model equations for the prediction of the dynamics of avalanching flows of dense granular materials down arbitrarily curved and twisted channels. The model equations describe the distribution of the avalanche thickness and the topography-parallel velocity components. These equations constitute a set of hyperbolic partial differential equations. A nonoscillatory central (NOC) differencing scheme with total variation diminishing (TVD) limiters is used to solve these equations. This high-resolution numerical technique is able to resolve steep heights and velocity gradients, often observed in experiments and field events, but not captured by traditional finite difference schemes (Pitman et al.,8,9 McDougall and Hungr,10 Patra et al.,13 Mangeney et al.,14,15). Pudasaini et al.16 used the model equations for the prediction of the dynamics of dry granular materials down simply and one-dimensionally curved chutes. Furthermore, numerical simulations for flows down uniformly and nonuniformly curved and twisted channels and channels with and without runout zones are extensively presented by Pudasaini et al.17

In order to acquire confidence in the model equations, it is vital to corroborate results obtained by them by direct

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observation for flows down different topographic configurations.\textsuperscript{6,18–25} For this reason, in the present paper we focus attention on some benchmark and pioneering laboratory experiments of dry granular flow avalanches over curved and twisted channels and comparison of the results with theoretical predictions. We performed several laboratory experiments with different granular particles in order to check the validity of the theory. We also present results for different channel types. We are particularly interested to analyze the deposition fans in settlements. The reason is that the correct determination of the depth profile of the deposit is very important in real applications.\textsuperscript{8–11,13} If there is a good agreement between theory and experimental measurement of the runout distance, areal coverage, and distribution of the height in the deposit, then one can infer the reliability and efficiency of the theory over the entire avalanche path. In this way the theory can be used to predict the evolution of the depth profile and other relevant physical quantities as the dense granular mass slides down the twisted channel or corridor. For these reasons, we have very carefully measured the depth profile of the deposit of the avalanche using an XY-table and analog laser sensor, a laser altimeter measurement technique. This method is highly accurate.

One of the major outcomes of this paper is the disclosure of the fact played by topographic parameters such as the twist and curvature on the dynamics and deposits of granular flows over a generally curved and twisted channel. For such a channel, it has been shown, both by simulation and experimental results, that these geometric parameters play a dominant role over physical parameters and that deposits in the runout zone are shifted, rotated, and specially curved to the left of the center line of the channel, thus presenting asymmetric deposits about the center line of the channel. This is a fact which was never investigated before, and we were able to present such results by using specially designed model equations which are written in a very natural coordinate system, which explicitly and genuinely include the curvature and twist of the channel. To check the validity of the theory we designed a new avalanche experiment, not considered before for such channel configurations. We are able to demonstrate that, viewed from the complexity of the problem, there is generally good agreement between the theoretical predictions of the model equations and the experimental measurements providing the applicability of the theory for granular flows down generally curved and twisted channels.

II. MODEL EQUATIONS

A. Role of topographic parameters

We want to study the effect of topographic parameters on the motion of a granular avalanche down generally curved and twisted channels. Such a study is important because transverse shearing and cross-stream momentum transport occur when the topography obstructs or redirects the motion due to its curvature and torsion. Local deceleration and deposition of mass may occur due to energy dissipation. Resistance due to basal friction is modified by "centrifugal forces" induced by bed curvature and torsion.\textsuperscript{2}

We consider the twisted channel that allows identification of the avalanche track along its talweg. A space curve parallel to this talweg is singled out as a master curve $C$ from which the track topography can be modeled, see Pudasaini and Hutter.\textsuperscript{1} The curvature and torsion of the master curve, $\kappa=\kappa(x)$, $\tau=\tau(x)$, can be computed from the elevation data of the channel as functions of the arc length $x$ of the master curve. Then, an orthogonal coordinate system along the talweg is introduced and the model equations are derived in this general coordinate system. In the model equations of this paper, $(x,y)$ form a curved reference surface, where $x$ is the coordinate along the talweg of the channel, while $y$ is the circular arc length in a cross-sectional plane perpendicular to the talweg of which the value is determined by the relation $y=\varepsilon z_T$; here $\varepsilon$ is a typical aspect ratio between the avalanche height and extent, $\theta$ is the azimuthal angle which accounts for the cross-slope curvature, and $z_T$ (usually $z_T \gg 1$) is the radial distance between the master curve and the talweg, while $z$ is the coordinate perpendicular to the reference topography. These quantities are written in nondimensional form. The channel topography and the geometry of the avalanche in the lateral and longitudinal directions are illustrated in Fig. 1 for a simplified case.

The theory is designed to model the flow of avalanching debris over channels having arbitrary curvature and torsion. Although there are other models that consider the problem of avalanche motion over curved slopes (e.g., Gray \textit{et al}.,\textsuperscript{3} Denlinger and Iverson,\textsuperscript{4,5} Kerswell,\textsuperscript{6} Ancey,\textsuperscript{7} Pitman \textit{et al}.,\textsuperscript{8,9} Maeno and Nishimura,\textsuperscript{26} Norem \textit{et al}.,\textsuperscript{27,28} Savage and Nohguchi,\textsuperscript{29} Savage and Hutter,\textsuperscript{30} Iverson \textit{et al}.,\textsuperscript{31} see also the same report edited by Harbitz,\textsuperscript{32} Bouchut and Westdickenberg,\textsuperscript{1} and Tai and Kuo,\textsuperscript{34})\textsuperscript{1} the model equations considered in this paper are the first to explicitly include curvature and torsion effects in a systematic manner (see Pudasaini \textit{et al}.).\textsuperscript{16,17,35} This makes the extended model amenable to realistic granular flow motions down arbitrary guiding topographies or transportation of granular masses in general channels in industrial process engineering.
B. Field equations

In this section we will outline the model equations proposed by Pudasaini and Hutter. They describe the dynamics of avalanching granular materials down general slopes and channels. The final thickness-averaged nondimensional balance laws of mass and momentum in (slope-fitted) curvilinear coordinates of channel surfaces in the down-slope and cross-slope directions take the forms

\[ \frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(hu) + \frac{\partial}{\partial y}(hv) = 0, \]  
\[ \frac{\partial}{\partial t}(hu) + \frac{\partial}{\partial x}(hu^2) + \frac{\partial}{\partial y}(huv) = hs_x - \frac{\partial}{\partial x} \left( \frac{\beta_x h^3}{2} \right), \]  
\[ \frac{\partial}{\partial t}(hv) + \frac{\partial}{\partial x}(huv) + \frac{\partial}{\partial y}(hv^2) = hs_y - \frac{\partial}{\partial y} \left( \frac{\beta_y h^3}{2} \right), \]

where \( h \) is the depth of the avalanche, \( u, v \) are the depth-averaged velocity components parallel to the reference surface, and the factors \( \beta_x \) and \( \beta_y \) are defined, respectively, as

\[ \beta_x = -e \kappa K_x, \quad \beta_y = -e \kappa K_y, \]

where \( e = H/L \) is the aspect ratio and \( K_x \) and \( K_y \) in Eq. (4) are the earth pressure coefficients. Elementary geometrical arguments in Mohr’s circle diagram of Coulomb frictional material may be used to determine these values as functions of the internal (\( \phi \)) and basal (\( \delta \)) angles of friction. \( K_x \) and \( K_y \) may have two values depending on the state of flow: The active state, during dilatational motion and the passive state, during compressional motion. The net driving accelerations in the down-slope and cross-slope directions are represented by the terms \( s_x \) and \( s_y \), respectively, and are given by

\[ s_x = g_x - \frac{u}{|\mathbf{u}|} \tan \delta( -g_x + \lambda \kappa \eta u^2 ) + e g_x \frac{\partial b}{\partial x}, \]
\[ s_y = g_y - \frac{v}{|\mathbf{u}|} \tan \delta( -g_y + \lambda \kappa \eta v^2 ) + e g_y \frac{\partial b}{\partial y}, \]

in which \( |\mathbf{u}| = \sqrt{u^2 + v^2} \) is the magnitude of the velocity field and \( g_x, g_y \) are the components of the gravitational acceleration along the coordinate lines. Similarly, \( \lambda = L/R \) is a measure of the radius of curvature \( R \) of the talweg of the bed with respect to the avalanche length \( L \). The aspect ratio \( e \) and the measure of curvature relative to the typical avalanche length \( \lambda \) are both assumed to be small. The basal surface, defined as the deviation of the basal topography from the reference surface \( z = 0 \) describing small-scale geometric features of the bed topography, will be denoted by \( z = b(x, y) \). Moreover, \( \lambda \kappa \) is the local radius of curvature of the talweg, while

\[ \eta = \cos \left[ \theta + \varphi(x) + \varphi_0 \right], \]

where \( \varphi(x) = -\int_0^x \tau(x')dx' \) gives the accumulation of the torsion of the talweg from an initial position \( x_0 \) and \( \varphi_0 \) is a constant. The torsion \( \tau \) is a function of the arc length and is calculated by taking the arc rate of change of binormal of the center line of the channel. In contrast to the plane flow configuration, the gravity components along the flow lines are calculated by considering the corresponding tangent, normal, and binormal vectors of the channel center line (see Secs. 4.3, 4.4, 9.2, and 9.4 in Pudasaini and Hutter). Also note that the advantage of the twisted coordinates over the plane flow configuration is that we do not require the explicit knowledge of the channel slope. The slope is automatically defined by the curvature and the twist. However, if needed, the slope of the twisted channel can intrinsically be defined by the ratio between the torsion and the curvature. This implies that, in the twisted channel, there is no singularity. For how to calculate the curvature and the twist, we refer to Sec. 4.3 in Pudasaini and Hutter.

The first terms on the right-hand sides of Eqs. (5) and (6) are due to the gravitational accelerations in the down- and cross-slope directions, respectively. The second terms emerge from dry Coulomb friction including the geometric effects via the curvature (\( \kappa \)) and torsion (included in \( \eta \)) of the channel, and the third terms are the topographic variations along the two flow directions. The last terms on the right-hand sides of Eqs. (2) and (3) are due to the pressure gradients induced by the free surface of the flow and associated earth pressure coefficients.

It is very important to note here that the role of \( \eta \) is crucial in these model equations in the description of the dynamics of the granular avalanche down general channels. This aspect is explicitly made clear by Pudasaini et al., with numerous simulation results, both for dry and water saturated granular and debris flows. If we set \( \eta = 1 \) then the channel is only one-dimensionally curved in the downhill direction so that the flow will always be symmetrical with respect to the center line or the talweg if the topography of the channel is symmetric with respect to it. Including \( \eta \) in the model equations, the channel is twisted so that the flow down such a channel experiences the centrifugal force associated with the twist of the channel. Therefore, depending on the intensity of the twist the mass will deviate from the center line and will be pushed to the channel side demonstrating an “overbanking” effect and thus remains asymmetric. Another important aspect of these equations is the inclusion of the detailed geometric effect of the channel. In the sequel, we will show that the shape, geometry, position, and runout distance of the avalanching granular mass in the deposition zone will be dominated by the form of the opening mouth of the channel in the transition into the runout. Depending on the opening curvature and twist of the channel the mass will be deposited to the outer corner or the inner corner of the channel or may also remain in the vicinity of the center line of the channel (Figs. 7, 9, and 11 to follow). So, both the channel form and the twist of the channel will influence the dynamics of the avalanching granular flows. For detailed studies of such effects we refer to Pudasaini and Hutter.

Equations (1)–(3) contain three unknown field quantities, namely, \( h, u, \) and \( v \). For a prescribed reference surface, basal topography \( b \), and material parameters \( \delta \) and \( \phi \), these equations can be solved as functions of space and time once appropriate initial and boundary conditions are prescribed. Note that the \( x \)- and \( y \)-coordinates are following the downhill
and cross-slope directions, respectively. In all results presented in this paper these two coordinate lines are directly used to form a three-dimensionally curved and twisted channel, with or without transition zone which may or may not be opened to merge into a horizontal plane. This is one of the major advantages of the model equations presented in this section. Therefore, in this paper we do not need to consider the third terms on the right-hand sides of Eqs. (5) and (6). In other, previous model equations it was not possible to form a three-dimensionally curved and twisted channel by using only the x- and y-coordinates (see Gray et al.,3 Denlinger and Iverson,4,5 Denlinger and Hungr,10 Patra et al.,15 and Mangeney et al.14,15).

C. Numerical technique

Equations (1)–(3) comprise a hyperbolic system of partial differential equations in three variables, the avalanche thickness and velocity components in the down-slope and cross-slope directions. Numerical schemes solving these equations must be able to grasp the typical behavior of such granular flows. Shock formation is an essential mechanism in granular flows on an inclined surface merging into a horizontal runout zone or encountering an obstacle when the velocity becomes subcritical from its supercritical state. It is therefore natural to employ conservative high-resolution numerical techniques that are able to resolve the steep gradients and moving fronts often observed in experiments and field events but not captured by traditional finite difference schemes. There are many alternatives to achieve this (see Denlinger and Iverson,4,5 Pitman et al.,8,9 Denlinger and Hungr,10 Patra et al.,15 Mangeney et al.,14,15 and Iverson et al.35). We use the NOC differencing scheme with the Minmod TVD limiter; it demonstrates superb numerical performance for simulating avalanche dynamics. This numerical scheme is extensively discussed and implemented for granular and debris flows down different topographic configurations in Pudasaini et al.,16,17,35 and Tai et al.36 Therefore, we do not elaborate on it here. We refer to Pudasaini and Hutter2 for detailed and extensive discussion on these and alternative numerical techniques and results for different configurations.

III. VALIDATING THE THEORY

The aim of this paper is the validation of Eqs. (1)–(3), presented in Sec. II, for motions of granular avalanches down twisted channels by judiciously designed adequate laboratory experiments. In particular, we will demonstrate, given the very complex nature of the problem, that there is a rather good agreement between theoretical predictions and experimental results for the runout distance, areal, and depth distribution of the deposit in the transition and in the runout zone of granular avalanches down nontrivially curved and twisted channels. Although these comparisons seem to be more qualitative than quantitative ones, they are nevertheless able to present a broad applicability of the theory and to show that the twist parameter \( \eta \) affects the flow dynamics of granular materials down curved and twisted channel.

FIG. 2. (Color online) Short twisted channel: Made of 5 mm Plexiglas, upper part is straight and inclined at (45°); the middle part is twisted and the lower part consists of transition and runout zones merging into the horizontal plane. Both the upper and lower parts are cylindrical with radius of 9.7 cm. Total length=419 cm; upper straight part=44 cm; middle twisted part=225 cm; lower transition portion=50 cm; and horizontal flat zone=100 cm.

A. Experimental arrangement

Several series of experiments were performed over two twisted channels, called the “short twisted channel,” shown in Fig. 2 and the “long twisted channel” shown in Fig. 3. Figure 4 depicts closeups of the long channel at four different parts of the channel. The channels are made of 5 mm thick Plexiglas. Both channels consist of four different parts with several portions connected together: The upper inclined straight part with inclination angle of 45°, the middle twisted

FIG. 3. (Color online) Long twisted channel: Upper part is straight and inclined at 45°; the middle part is twisted and the lower part consists of transition and runout zones merging into the horizontal plane. Both the upper and lower parts are cylindrical with radius of 9.7 cm. Total length of the channel=600 cm; upper straight part=85 cm; middle twisted part=365 cm; lower transition part=50 cm; and horizontal flat part=100 cm.
part, the lower open transition zone, and the final flat horizontal runout zone. The short twisted channel is \(419\) cm long. Similarly, the long channel is \(600\) cm long. For both short and long channels, all channel configurations before the transition region have circular cross-sectional geometry with radius of \(9.7\) cm. The talwegs of both channels are measured. Figure 5 shows the talweg of the short channel.

Experimental images are taken for different granular materials and chute geometry configurations in the transition and runout zones. At the top of the first inclined element a vertical shutter is mounted to hold the initial granular mass. The shutter is manually lifted vertically to let the mass of the granules slide down. Three different granular materials, silicon dioxide, brown quartz, and crystal sand, are used for the experiments.

**Choice of different channels and materials**

We would like to mention at this point that we have several reasons for choosing different channel lengths and materials for the experiments. First, if the channel was only curved and not twisted the deposition geometry would, strictly, be symmetrical with respect to the central line of the channel (see Gray et al.,\(^3\) Iverson and Denlinger,\(^4\) Pudasaini et al.,\(^16\) and Hutter et al.\(^37\)). A large amount of research has been done for this type of particular rolled surface; thus, we will not report here on these particular and simpler cases. A collective review of results can be found in Pudasaini and Hutter.\(^2\) However, experiments for the flow of granular materials down a generally curved and twisted channel were lacking in the literature as were their comparisons with a suitable theory, in which the coordinates are exactly following the channel. This is the main focus of this paper.

Since the long channel is considerably longer than the short one, effects of the curvature and torsion will be more pronounced for it. As a test example, we used the same amount of material of the same type for the long and short channels and wanted to observe the effect of the curvature and twist as geometric parameters on the dynamics of the flow. One would expect a larger influence of the twist on the long channel than on the short channel. We are also interested to investigate the locus and shapes of depositions for experiments over the long and short channels. Even with knowledge of the effect of the centrifugal force, one would expect that, depending on the amount of the twist and channel length, the deposited body may lie more on the inner half portion of the channel because the longer the channel the more the accumulated twist. The influence of the twist of the channel can also be reflected in the form and the shape of the deposition, giving indication that the body would be rotated from the center line toward the inner boundary of the channel, by a significant amount, all showing the strength of the twist on the dynamics. The fundamental effects of the channel geometry, mainly the curvature and twist, will be manifested in the position, lateral and longitudinal extents and depth distribution of deposits, and curvature of the free surface. Similarly, the detailed differential geometric features of the chute, immediately above the deposition zone, strongly influence the geometry of the deposited mass.

A further aspect of this paper is the use of different materials for three experiments: Silicon dioxide, brown quartz, and crystal sand grains. These experiments also differed by the initial total masses and material parameters, such as the internal and basal friction angles. Together with the geometry of the channel, these material parameters reveal their influences on deposits. However, it is found that geometrical parameters are dominant over material parameters. A fact seldom discussed in the dynamics of granular flows because research on the dynamics of granular flows are still largely based on the classical theories and experiments on straight and simply curved chutes, which could not explore these strong geometrical features of the channel. For this reason, this paper demonstrates new aspects of the research of granular flows.
TABLE I. Geometrical and physical parameter values involved in different experiments and the extent ratio in the deposit. The sign ± indicates the error in the measurements of corresponding physical parameters.

<table>
<thead>
<tr>
<th>Exp. No.</th>
<th>Channel length (cm)</th>
<th>Material type</th>
<th>Mass (kg)</th>
<th>φ (°)</th>
<th>δ (°)</th>
<th>Mean particle diam. (mm)</th>
<th>Extent ratio (×)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>419</td>
<td>Silicon dioxide</td>
<td>2.78</td>
<td>35 ± 2.5</td>
<td>25 ± 2.0</td>
<td>1.8</td>
<td>2.2 ± 0.05</td>
</tr>
<tr>
<td>2</td>
<td>600</td>
<td>Brown quartz</td>
<td>2.78</td>
<td>30 ± 2.0</td>
<td>20 ± 1.5</td>
<td>1.6</td>
<td>2.0 ± 0.06</td>
</tr>
<tr>
<td>3</td>
<td>545</td>
<td>Crystal sand</td>
<td>3.75</td>
<td>44 ± 3.0</td>
<td>25 ± 2.5</td>
<td>5.0</td>
<td>2.0 ± 0.04</td>
</tr>
</tbody>
</table>

B. Evolution of granular flow geometry

1. Short channel experiments

An important aspect in avalanche dynamics of granular flows is the determination of the travel distances, runout areas, and height profiles in the deposit zone. For this purpose, different (laboratory) techniques such as laser light sheets, laser cartography, digital photography, or digital photogrammetry must be used (see Pouliquen and Forterre and Iversen et al.). For a review of different measurement techniques, we refer to Pudasaini and Hutter. In this paper, we measured the depth profiles in deposits using an XY-table and analog laser sensor (SensoPart Inc.), a laser altimeter-measurement technique.

Experiment 1: Experiments with silicon dioxide. First we discuss the measurement results of the sliding and deforming granular avalanche through the short channel. The details of the used material are as follows: 95% white silicon dioxide +5% (as weight fraction) dark tracer particles with a diameter of 1.8 mm. The mass of the bulk material is 2.78 kg. The internal and bed friction angles of the mixture are φ=35° and δ=25°, respectively. The measurement techniques for the determination of the internal and bed friction angles are mentioned in detail in Pudasaini and Hutter and Greve and Hutter. In Table I, we have mentioned the measurement errors for these physical parameters.

Figure 6 depicts the image of the deposited granular material in the transition and horizontal runout zones. The extents of the settled body in the longitudinal and transversal directions are 55 and 25 cm, respectively. Due to the gradual opening of the channel in the lateral, and the decreased curvature in the down-slope directions, the mass is considerably extended also in the lateral direction after the avalanching granular body crosses the middle twisted part. Before that the mass is mainly extended in the longitudinal direction due to the confining effect of the channel. The avalanche body is shifted a bit to the left of the talweg when looking from the front. The reason for this is that the channel in the middle part is twisted which introduces the centrifugal force into the motion. This forces the bulk to head toward the outer boundary of the channel. However, the channel geometry is such that in the vicinity of the upper boundary of the transition zone the twist of the channel suddenly drops to zero and the curvature also gradually decreases to zero, while the lateral curvature falls only slowly and gradually to zero. Therefore, in such a situation the gravity component toward the talweg and the outer boundary traction cause the bulk to cross the center line (the talweg) of the channel and move to the other side of the channel in the transition zone. So, mass in its deposition is located slightly toward the inner boundary of the channel.

Figure 7 displays the boundary of the settled mass in the deposition area for the theoretical prediction (right panel) and, respectively, the experimental outline of the margin (left panel). The locations of the experimental and predicted boundaries of the granular mass in the settlement area are in fair agreement to one another.

We would also like to mention here that in the simulation the contour with the mean particle diameter is chosen to determine the boundary of the deposit. In the experiment, it is the locus on the channel surface such that 50% area is covered by the deposited grains. These methods are explained by Pudasaini and Hutter and Koch et al. As an example, Fig. 10 shows its original form. This shows that the boundary is clearly defined in the major part of the deposition, mainly in the tail and along the sides of the deposit, but due to the possible particle agitation, bouncing, rolling, and ballistic motions, some particles in the frontal ring are not in contact with the main body (Pudasaini et al.).

2. Long channel experiments

Experiment 2: Experiments with brown quartz. We performed several experiments also with the long channel. First we consider the experiment with brown quartz. The material parameter values are mass of 2.78 kg, mean diam-

FIG. 6. (Color online) Top view of the deposition of the granular mass in the fanlike transition zone between x=270 and x=320 and runout zone of the short channel. The mass is shifted to the left of the talweg (towards the inner boundary) of the channel. The body has ellipsoidal form with its major axis parallel to the center line and the minor axis in the transversal direction. White particles (95% (weight fraction), appearing here in light brown) constitute the background material and the dark particles (5%) represent the tracer particles. Experiment was conducted with 2.78 kg silicon dioxide.

FIG. 7. Boundaries of the settled mass in the short channel experiments for the theoretical prediction (right panel) and, respectively, the experimental outline of the margin (left panel). The locations of the experimental and predicted boundaries of the granular mass in the settlement area are in fair agreement to one another.
eter of 1.6 mm, background particles of 95%, tracer particles of 5% (as weight fraction), and internal and bed friction angles of the mixture are $\phi = 30^\circ$ and $\delta = 20^\circ$, respectively. A typical deposit is displayed in Fig. 8. The results for the deposition for this case are shown in Fig. 9. The right and left panels represent, respectively, the predicted and measured boundaries of the settled granular mass. Since this channel is considerably longer than the short channel the effects of the curvature and torsion are more pronounced in this case. Although the amount of mass used for the experiment is the same as before the runout distance in this experiment is much longer than for the short channel. The deposition takes place almost exclusively in the horizontal runout zone. Here we focus our attention mainly on the geometrical parameter of the channel rather than the material properties of the grains used in different experiments. A very interesting point in this experiment is the location and shape of the deposition. As seen in the left panel the longitudinal and lateral extents of the settled body are about 100 and 50 cm, respectively. The corresponding values for the short channel were only 55 and 25 cm, respectively. The body lies almost completely on the inner half portion of the channel from the center line. A significant influence of the twist of the channel is reflected in the form and shape of the deposition. The body is rotated toward the inner boundary of the channel by about $25^\circ$ to the center line. More interesting is the curvature of the boundary in the lower left part of the figure. In this experiment, the middle part of the channel is sufficiently long to accumulate the twist ($\phi$ in the theory in Sec. II B), and it exercises its effect in the dynamics of the flow before the mass enters the transition zone. Then the twist suddenly drops to zero. All these special geometric features of the channel are physically transferred to the sliding mass. Due to the centrifugal force induced by the twist of the talweg, the body always heads toward the outer boundary of the channel before the mass reaches the transition zone. As soon as the body enters the upper boundary of the transition zone the confined channel strongly forces (the front part of) the body to cross the central line and move to the inner side of the channel. The continuously flattened lower portion of the transition zone again partially redirects the mass toward the central line of the channel; this is responsible for the special concave curvature of the boundary in this region as observed from the lower left part of the channel. At last, the mass is collected in the horizontal flat part of the channel where it

FIG. 7. (Color online) Boundaries of the deposits of the granular masses in the transition and runout zones of the short channel. Right panel: Predicted result. Left panel: Observed in the experiment.

FIG. 8. (Color online) View of the deposition of the granular mass in the fanlike transition in the horizontal runout zone of the long channel. The mass is shifted and deflected to the left of the talweg (toward the inner boundary) of the channel. The body has a “mango-type” form with its major axis inclined about an angle of $25^\circ$ to the center line and the minor axis with similar inclination in the transversal direction. Flow direction is from top to bottom as indicated by the arrow.
looses all the history of the curvature and twist which is obviously reflected in the deposit. The right panel represents the predicted result of the theory. Although the location of the front of the simulated deposit is somewhat farther than the experimental one and the transversal extent is somewhat smaller, the overall effects of the geometric parameters of the channel are in convincingly good agreement with the left panel.

**Experiment 3: Experiments with crystal sand.** Finally, we consider the experiments with crystal sand with the following material and parameter values: Mass of 3.75 kg, mean diameter of 5 mm, background dark particles of 90%, tracer white particles of 10% (as weight fraction), and friction angles of the mixture $\phi=44^\circ$, and $\delta=25^\circ$, respectively. For the experiments with crystal sand we installed the analogaser sensor in the $XY$-table so as to acquire accurate knowledge of the depth distribution in the deposit and thus to check the overall quality of the theoretical prediction.

One very important aspect of the granular and debris flow is its deposit or depth distribution in the runout zone, mainly in the floor of the deposit in the form of fans. From this information, one can study many different phenomena in the breaking zone and along its track. So, for this reason, we measured the depth distribution of the avalanching debris flow in the fanlike transition and horizontal runout zone. Figure 10 depicts the deposit of 3.75 kg of crystal sand where background particles are dark and tracer particles are white. The transversal thick dark line below the center marks the continuous transition to the horizontal from a gentle cross-sectional parabolic channel. In this picture, the left edge is the inner boundary of the channel and the right edge is the channel outside boundary.

In Fig. 11, predicted depth contours of the deposited granular mass are plotted in the right panel, whereas their observed counterparts are shown in the left panel in the transitional and horizontal runout zones of the long channel. The position, lateral and longitudinal extents, and the depth distribution of the deposits manifest overall effects of the curvature and twist of the channel. As in the case of brown quartz in Fig. 9, they have significantly long runout extents in both the longitudinal and transversal directions and large areal spreading. Both panels are rotated, twisted, and shifted to the left part (toward the inner boundary) of the channel, both panels have concave curvature of the boundary when viewed from the left corner of the runout zone. The arrows indicate the center line of the channel. These lines are the basis for the description of the dynamics of the flow and deposit. Here, it is very useful to gain an overall idea about the deflection, shift, and rotation of the deposited mass from the center line of the channel. If the channel were only curved and not twisted the deposition geometry would, strictly, be symmetrical to the center line (see Pudasaini and Hutter and Pudasaini et al.). Note that the channel lengths for this experiment and the experiment in Fig. 9 are not the
same. For the experiment with crystal sand one portion in the middle part (55 cm) is removed so reducing the total length of the channel which is also manifest in the shorter travel distance in both experimental and numerical panels. Another fact is that the material parameters for crystal sand particles are different from those of brown quartz. Crystal sand particles are much coarser (5 mm mean diameter) than the particles with mean diameters of 1.8 and 1.6 mm for silicon dioxide and brown quartz, respectively. This difference is manifested in the internal and bed friction angles. So, for the present case there is less proneness for slip and areal extent in the runout zone. This fact is clearly revealed by comparison of Figs. 9 and 11. However, since we are using continuum theory for the theoretical prediction of the granular mass flow the areal extent in the theoretical panel is somehow larger than in the experimental panel. This areal spread of the mass is counterbalanced by the fact that the height of the three patches in the core of the deposit of the panel on the right is smaller in comparison to their respective patches in the experimental panel on the left which can be realized by comparing the color codes of the respective panels. However, in general, with respect to the complex nature of the channel, the agreement between the experimental and predicted results is judged as fair.

C. Role of geometric and physical parameters

The geometrical and physical parameters involved in all three experiments are collected in Table I which consists of the channel lengths, initial masses, material types, internal and bed friction angles of granular materials with respect to the roughness of the sliding channel, and the mean particle diameters. As explained in the previous section, the role of the physical and geometrical parameters is clearly seen in deposits. For longer channels (experiments 2 and 3) the role of the twist is more pronounced so that the final deposit takes place mainly in the inner part of the channel center line and deposits are rotated considerably. However, for the short channel (experiment 1), the deposited mass was only slightly shifted to the left but not rotated in the runout zone (both as observed from the front-top view). For the short channel, due to the limited channel elevation and associated twist, the final deposit was close to the mouth of the runout zone, so the lateral extent of the deposit was relatively small.

The last column of Table I shows values for the extent ratio (which is the ratio of the longitudinal; as measured along the center line of the channel; to lateral extents of the deposits). It is interesting to analyze this ratio. This clearly reveals that, although other physical parameters are also important in the description of the flow dynamics, the major role is played by the channel twist and the associated length. For experiments 2 and 3, this ratio is almost 2, while for experiment 1, it is a bit larger. Because, as explained earlier, for the short channel the deposited mass is very close to the channel mouth so it could not sufficiently spread in the transversal direction to keep this extent ratio low. Although channel lengths for the two long channels and other physical parameters were not exactly the same, including the initial masses, the extent ratio is surprisingly constant, thus showing the strong effects of geometrical parameters, such as the channel twist and curvature.

IV. MEASUREMENT ERRORS

All experiments were performed under a well controlled laboratory environment. Keeping the same initial and boundary conditions, all three different experiments were performed several times in order to check the reproducibility of the particular type of experiment. After each experimental run, the channel was cleaned with an antistatic spray gun so as to keep the electrostatic force, induced by the friction between flowing grains and the Plexiglas channel at low level.
The maximum resolution of the analog laser sensor is 0.1 mm. About the measurement error of the particle diameter: The material is almost free of small grains and powder. The mean value for the diameter was determined by manually measuring the diameters of randomly chosen 35 particles in each type of experiments. Measurement errors on the friction angles and the extent ratios are mentioned in Table I.

One major source of measurement error for the determination of the boundary of deposition was the spreading of some grains outside the continuum boundary. This was found to depend on the amount of the mass used for the experiment and also the length of the channel as well as the internal and bed friction angles of the granular material. When the amount of mass used was high, then the relative amount of the particles spreading outside the continuum boundary was low, while on the other hand, for the longer channel, particles were more agitated during the course of the flow as they had to travel longer distances. This resulted in a bit larger spreading of particles outside the continuum boundary of the deposit. However, the amount of particles leaving the boundary of the main deposited body was very low, below 1% of the total initial volume used for each run, as can be inferred from Fig. 10.

Next, the front and the rear positions and also the extreme lateral reach of the deposition were considered. We could almost exactly reproduce the deposit of the mass in each repeated experiment for all three experiments within an error limit of 0.6% of the total depositional extent in both lateral and longitudinal limits. This error limit was obtained by taking the standard deviation from the mean of the three similar well controlled experiments of each type.

Yet, another source of error was observed in the measurement of the boundary of the deposits (experiments 1–3) and depth contours of the deposit (experiment 3). For experiments 1 and 2 the marginal limits of the deposits were measured based on the known coordinate line (indicated by dots on the channel surface in the runout and deposition zone, see Fig. 6) spacing, a few centimeters apart. These boundaries were mapped very accurately, within 0.5% of the relative error of the total extent, by mapping the exact boundary from the demarcation acquired from the channel, then again mapped to a sheet of a paper.

The final measurement error was related to depth contours in experiment 3. This concerns the accuracy of the analog laser sensor mounted in the XY-table. The advantage of this method is that this is a noninvasive and highly accurate technique for the determination of elevation of the deposited mass. For this, we first calibrated the system by carefully measuring the altitude of the runout zone as reference altitudes of points in the centimeter grid. The exact altitudes of these points were already known from the construction of the channel. We observed that those reference values were very accurate. The accuracy of the measuring device is about 0.5 mm. Once the system was calibrated, experiments were performed and altitudes of the free surface were measured exactly at those grid points as employed in the calibration of the measurement of the runout zone. Then, finally, the depth distribution was determined by taking differences between those altitudes of the free surface and corresponding reference altitudes of the basal sliding surface in the runout and deposition zones.

V. SUMMARY

We reported about three laboratory experiments of the flow of finite-mass-granular avalanches down curved and twisted channels from high altitude initiation through a semi-circular duct into a flat deposition zone. The materials in the three experiments were silicon dioxide and brown quartz and crystal sand grains, and the experiments differed by the applied total masses, the internal and basal friction angles, and different chute geometries.

Here, we focused on the distance traveled by the granular masses and the positions, geometries and mass distributions within the deposits. XY-tables and analog laser sensors were also used to determine these. It was found that the channel length, the curvature and twist of its talweg, the geometry of the mouth of the channel in the transition zone, as well as the frictional parameters, expressed as internal and basal friction angles, affect the characteristics of the deposits.

The motions of the granular masses through the laboratory chutes were also numerically simulated, using the model equations developed by Pudasaini and Hutter. Comparisons of the experimental and numerically predicted deposition boundaries as displayed in Figs. 7, 9, and 11 illustrate relatively good coincidence of the marginal boundaries, Figs. 7, 9, and 11, and mass distributions, Fig. 11. The experiments also indicate that the differential geometric details of the chute in the immediate vicinity above the deposition zone influence the geometry of the deposited mass. If the channel elements in the last stretch of the chute are torsion-free, then “centrifugal effects” are no longer effective and the settling mass is pushed by the boundary tractions from the chute outside to the chute inside, and the deposit is eventually elongated and essentially parallel to the torsion-free talweg, Fig. 7. If, on the other hand, the material passes through a rather longer chute with twist that decreases further downstream, then the effect of the centrifugal forces also decreases in the downward direction while the pressure at the outer boundary gains growing significance. This leads to an apparent rotation of the deposited mass relative to the talweg, Figs. 9 and 11.

The role of the physical and geometrical parameters is clearly seen in the deposits. For the long channels the role of the twist was more pronounced so that the final deposit took place mainly in the inner part of the channel center line and deposits were rotated considerably. However, for the short channel the deposited mass was only slightly shifted to the left but not rotated in the runout zone. Such asymmetric depositions and surface contours could not be predicted by any other classical theories, and also no experiments were available in literature. Simulations from other theories would only produce symmetrical depositions about the center line of the channel. This manifests strong effects of geometrical parameters, such as the channel twist and curvature, on final deposits and also the position and shape of the mass in standstill. Such a dominant role played by geometrical parameters.
of the channel over physical parameters for flows of granular materials down general channels was never investigated before. For this reason, we believe that our simulations and experimental results, which serve as benchmark problems for flows down general channels, are of particular importance.

Another important aspect is the extent ratio between the longitudinal and lateral reaches of depositions. Although channel lengths and other physical parameters including the initial mass were different for all experiments, the extent ratio is found surprisingly to be almost the same.

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